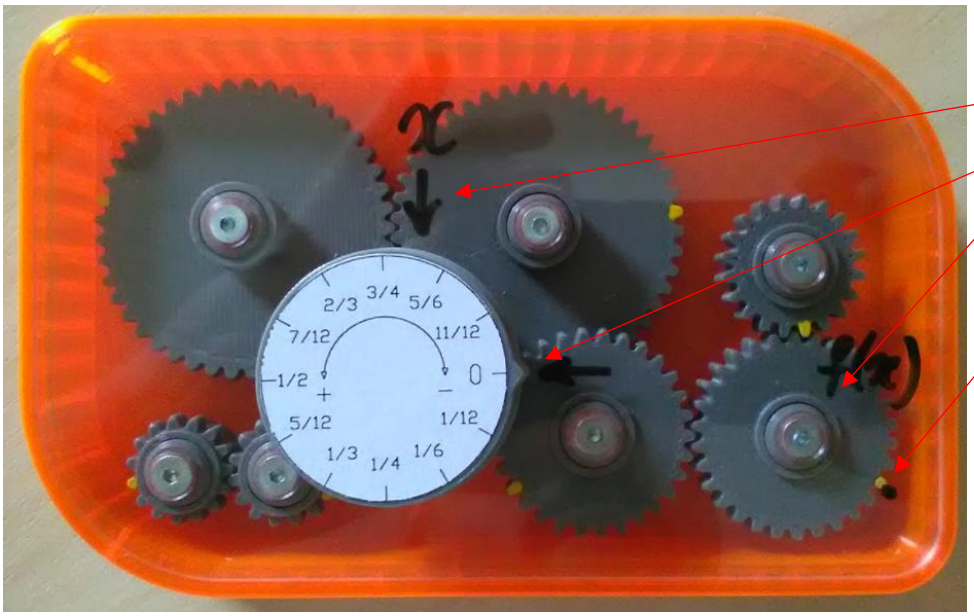
Slope of linear function

Linear function is the first mathematical function taught in K-12 school. Analysis of students' misconceptions – along with feedback from teachers show, that slope of linear function – or $\frac{\Delta y}{\Delta x}$ is very often not understood. Therefore, we developed simple, fully mechanical – yet very visual learning aid dedicated specifically for understanding the slope of linear function.

This artefact consists of primary gear – that corresponds to independent variable x, and 7 secondary gears (corresponds to dependent variable $f(x)$ or y). Gear x can be rotated with supplied knob. Use supplied marker to write on the front side of artefact. Marker can be erased with the sponge located on the marker cap.

Working with artefact 1 – linear slope for $y = x$

Presentation of artefact to students can be started with demonstrating functional relation between rotating primary and secondary gears. Let's agree, that rotation of knob counter clockwise corresponds to positive rotation. It can be seen in the picture, that one turn of x corresponds exactly to one turn of y. Two rotations of x correspond to two turns of y and so on.



Use marker to mark:

x

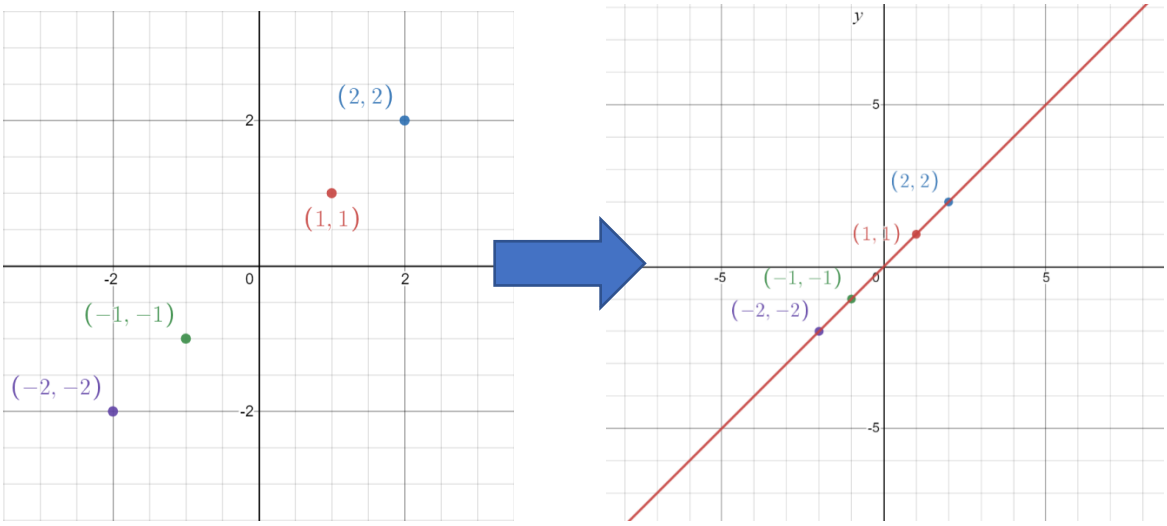
Zero point

$f(x)$ or y

Position of secondary gear

If we rotate x in opposite (clockwise) direction, let's call it negative rotation. -1 turns of x will correspond to -1 turns of y, -2 turns will correspond to -2 turns of y and so on.

Let's show it on the Cartesian coordinate plane:

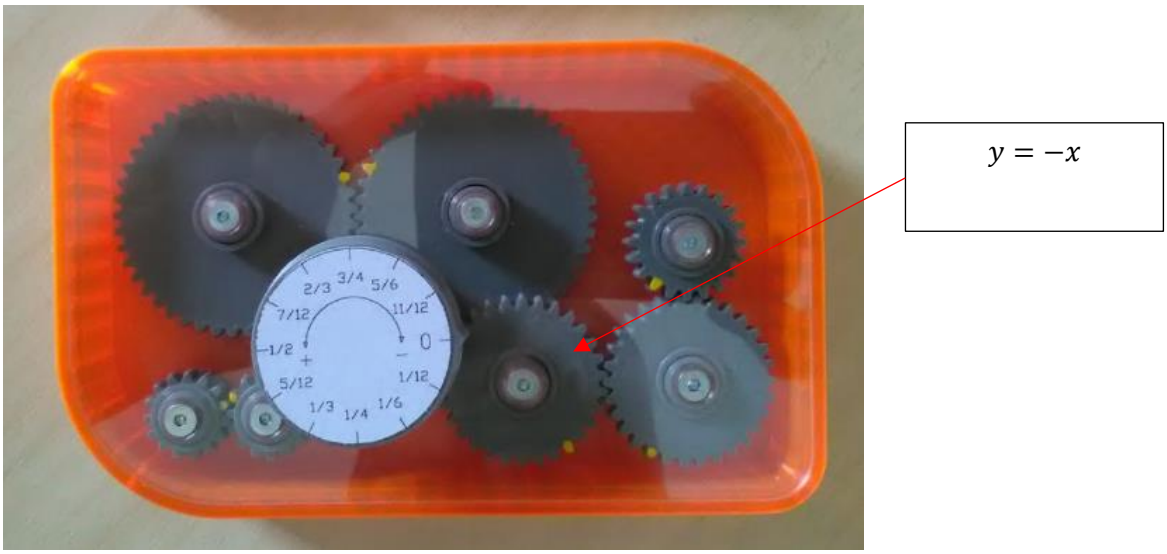


It can be seen, that number of rotations of y is always the same as number of x rotations. Moreover, it can be seen, that straight line can be drawn through all points. This kind of dependence is called linear dependence or linear function – when x is changed by 1, y is also changed by 1. This is called a slope of linear function and is calculated as $\frac{change\ of\ y}{change\ of\ x}$.

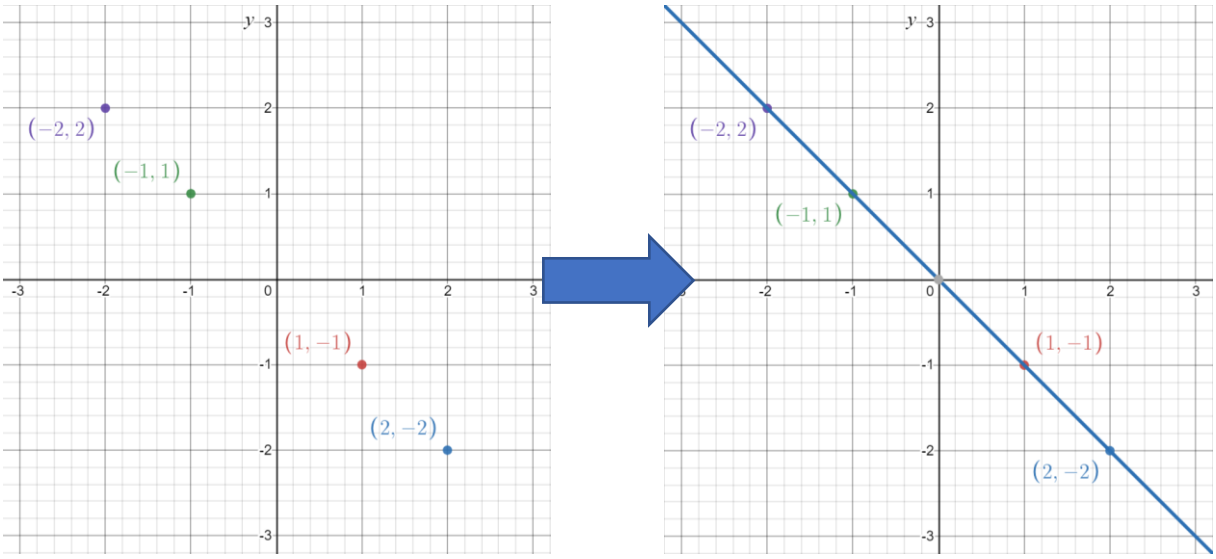
Question: how many turns will make gear y, if we rotate gear x 5 times? 100 times? 1000 times? 1/3 times?

<u>Working with artefact 2 – linear slope for $y = -x$</u>

Linear dependence can be different. Let's rotate x and look at another gear:



It can be seen, that for each positive rotation of gear x - gear $f(x)$ or y make one negative rotation (i.e. gears are rotating in opposite directions). It means, that for one x, there is one "$-y$" (minus y). And vice versa – for one negative rotation of x, there will be one positive rotation of y:

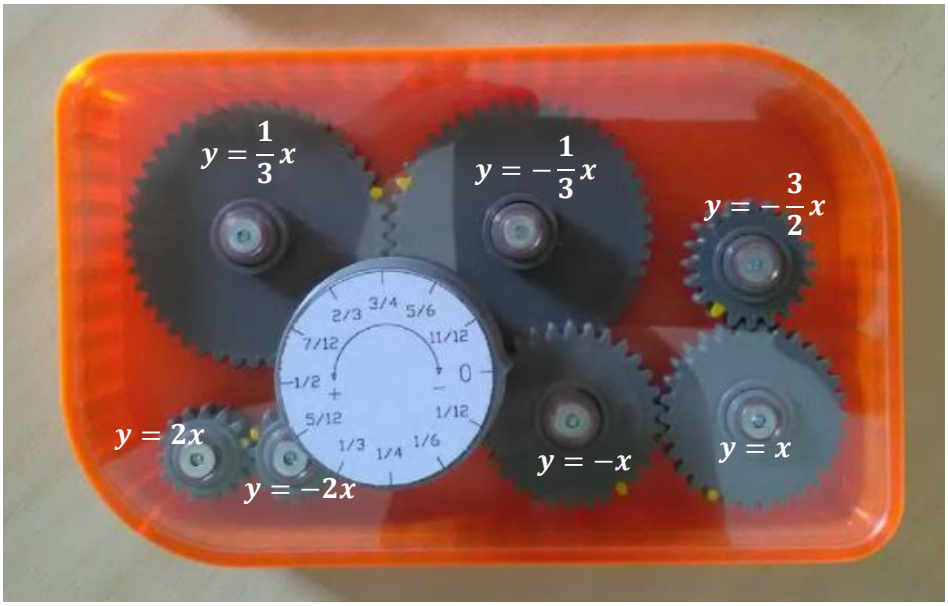


This kind of dependence can be written as $y = -x$: when x is changed by 1, y is changed by -1, also when x is changed by -1, y is changed by 1. The slope of function in this case is a $\frac{y}{x} = \frac{-1}{1}$ or $\frac{1}{-1} = -1$.

Question: how many turns will make gear y, if we rotate gear x 5 times? 100 times? 1000 times? 1/3 times?

Exercises for students:

Students can be now asked to experiment: rotate knob x and observe number of rotations of secondary knobs and therefore define slopes:



More difficult exercise

To define the slope of gear $y = -\frac{3}{2}x$. Hint to be given to students if necessary: this exercise can be solved in three ways:

- Rotate x two times – y will rotate – 3 times.
- Rotate x one time – y will rotate – 1.5 times.
- Rotate x and count until y will make 1 turn. The marking on the knob will show 2/3.

All three ways can be shown on the coordinate plane: